Decimal Expansion Of Rational Numbers

The real numbers which are recurring or terminating in nature are generally rational numbers.

Theorems

Theorem 1: If m be any rational number whose decimal expansion is terminating in nature, then m can be expressed in form of pq, where p and q are co-primes and the prime factorization of q is of the form $2^x X 5^y$, where x and y are non-negative integers.

The converse of this theorem is also true and it can be stated as follows:

Theorem 2: If m is a rational number, which can be represented as the ratio of two integers i.e. pq and the prime factorization of q takes the form $2^x x 5^y$, where x and y are non-negative integers then, then it can be said that m has a decimal expansion which is terminatin

Consider the following examples:

1. $\frac{7}{8} = \frac{7}{2^3} = \frac{7 \times 5^3}{2^3 \times 5^3} = \frac{875}{10^3}$
2. $\frac{3}{80} = \frac{3}{2^4 \times 5} = \frac{3 \times 5^3}{2^4 \times 5^4} = \frac{375}{10^4}$

Moving on, to decimal expansion of rational numbers which are recurring, the following theorem can be stated:

Theorem 3: If m is a rational number, which can be represented as the ratio of two integers i.e. pq and the prime factorization of q does not takes the form 2x 5y, where x and y are non-negative integers. Then, it can be said that m has a decimal expansion which is non-terminating repeating (recurring).

Solve Ex 1.4